Eigenfunction expansion of the dyadic Green's function in a gyroelectric chiral medium by cylindrical vector wave functions

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Gyroelectric chiral media, which blend gyroelectric effects with those of optical activity, have potential applications in chirality management. In the present investigation, based on the Ohm-Rayleigh method, the dyadic Green's function in an unbounded gyroelectric chiral medium is rigorously represented in an eigenfunction expansion of the cylindrical vector wave functions. The analysis reveals that the singularity of the dyadic Green's function is essentially static in character and independent of the chiral parameter. Due to the effects of optical activity, two types of eigenwaves, left- and right-handed circularly polarized waves traveling with different wave numbers, can be simultaneously excited in a gyroelectric chiral medium. Nonreciprocal property of the dyadic Green's function, owing to the gyroelectric effects, is revealed. It is found that the chiral effects of the gyroelectric chiral medium are manageable with the introduction of a controllable gyroelectric parameter to manage the wave numbers of the eigenwaves propagating in this class of medium. [S1063-651X(97)10302-6]

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I. INTRODUCTION

The direct solution method for source-free Maxwell's equations in an isotropic medium with vector wave functions was first proposed by Hansen [1] in the 1930s. This approach, which has been intensively developed by Stratton [2], Morse and Feshbach [3], and Tai [4] to solve the source-free and source-incorporated electromagnetic boundary value problems in isotropic media, has found increasing interest and importance. The vector wave functions have found versatile applications and present great advantages compared with other methods (e.g., the three-dimensional moment method [5], the coupled-dipole method [6], and the integral equation technique [7]) in solving both the source-free and source-incorporated electromagnetic boundary value problems for resonance, scattering, radiation, and propagation phenomena. For instance, the circular cylindrical vector wave functions have been successfully employed in studying the radiation characteristics of a dipole antenna in the proximity of a gyroelectric cylinder [8]. The dyadic Green's function is one of the basic tools that are used to solve the source-incorporated Maxwell's equations. It is useful both in analyzing radiation problems [4,8,9] and in constructing integral equations for scattering problems [10,11]. The general representation of the dyadic Green's function expressed in terms of an expansion of the vector wave functions is required to study Raman and fluorescent scattering by active molecules embedded in a particle [12,13], as well as to establish *T*-matrix formulation from Huygen's principle and the extinction theorem [14,15]. Furthermore, the eigenfunction expansion of the dyadic Green's function could also provide fundamental insight into the physical process for the material under consideration. However, much effort is still required in order to obtain the dyadic Green's function in any given complex material when expressed in the full eigenfunction expansion of the vector wave functions.

With recent advances in polymer synthesis techniques, increasing attention is being attracted to the analysis of interaction of electromagnetic waves with chiral medium in order to determine how to use this class of material to provide better solutions to current engineering problems [16–18]. It has been shown that the most important characteristics of chirality is that two types of circularly polarized eigenwaves could simultaneously propagate in a chiral medium, each traveling with a different wave number [16–18]. This class of material can be utilized to construct antireflection shields, novel reciprocal microwave components, and antenna radomes [16–18]. Therefore, chirality management, i.e., the management of the effects of chirality, seems to have potential applications in the control of the physical behaviors of devices that are made of chiral material, because the electromagnetic properties of chiral devices are directly connected with the chiral parameter. However, only limited methods exist for chirality management once the chiral material is created. One exception is through the introduction of certain forms of controllable anisotropy that can be realized either by employing the electro-optic and piezoelectric effects, or by introducing certain forms of externally biased controllable magnetic fields. With chirality management as the motivation for their investigations, Engheta, Jaggard, and Kowarz proposed the concept of a Faraday chiral medium [19] and examined the propagation characteristics of electromagnetic plane waves in an unbounded Faraday chiral medium that blends gyrotropic effects with those of optical activity. The reflection and transmission properties of electromagnetic waves through a Faraday chiral slab with finite longitudinal extent were investigated and the interplay between gyrotropic and chiral effects was studied [20]. Krowne [21] presented general formulations of a composite chiral-ferrite medium, including the dispersion relation, nonreciprocal properties, and polarization characteristics. Recently, field representations in a source-free gyroelectric chiral medium [22] as well as a composite chiral-ferrite medium

[23] have been presented in terms of the cylindrical vector wave functions based on a spectral angular expansion method. Nevertheless, much effort is still necessary in order to achieve a thorough understanding of chirality management.

A gyroelectric chiral medium, formed by immersing chiral objects in a gyroelectric material with arbitrary orientation, is a subset of the wider class referred to as bianisotropic media. In a gyroelectric chiral medium, the gyroelectric parameter may be managed through the introduction of an externally biased, controllable magnetic field. Important research on general bianisotropic media has been presented by Post [24], Kong [25], and Chen [26], among others. In contradistinction to these general considerations, the present contribution is intended to derive the eigenfunction expansion of the dyadic Green's function in gyroelectric the chiral medium in terms of the solenoidal as well as irrotational cylindrical vector wave functions. Another purpose of the present study is to investigate the physical process through which the gyroelectric parameter would have a significant effect on chirality management. These formulations are considerably simplified by using the Ohm-Rayleigh method [3,4] and introducing a set of linear combinations of the solenoidal vector wave functions. The formulations, which are standard and straightforward, lead to explicit expressions of the dyadic Green's function in an unbounded gyroelectric chiral medium. In order to make the present formulation available to practical applications, the contributions from the irrotational vector wave functions to the dyadic Green's function are intensively investigated, and the solenoidal parts are simplified by analytically evaluating the integrals with respect to the spectral longitudinal and radial wave numbers, respectively. It is found that the irrotational vector wave functions in the full eigenfunction expansion of the dyadic Green's function, which are essentially static in character, not only contribute at the source points, but outside the source as well. It is also indicated that, due to the chiral effects of the gyroelectric chiral medium, two types of eigenwaves, the left- and right-handed circularly polarized waves, can be simultaneously excited, each traveling with a different wave number. The fact that the wave numbers of these two eigenwaves are jointly determined by the gyroelectric and chiral constitutive parameters makes it possible to manage the chiral effects through the introduction of a controllable gyroelectric parameter. A nonreciprocal property of the dyadic Green's function in the gyroelectric chiral medium, which arises from the gyroelectric effects, is revealed. The present formulations can be verified by comparing their special forms with existing results corresponding to isotropic and chiral media.

In the following analysis, the harmonic $\exp(-i\omega t)$ time dependence is assumed and suppressed throughout.

II. GENERAL FORMULATION

From a phenomenological point of view, a homogeneous gyroelectric chiral medium can be characterized by the set of constitutive relations [19,22],

$$\mathbf{D}=\overline{\boldsymbol{\varepsilon}}\cdot\mathbf{E}+i\xi_c\mathbf{B}, \tag{1a}$$

$$\mathbf{H}=i\xi_c\mathbf{E}+\mathbf{B}/\mu, \tag{1b}$$

where $\overline{\boldsymbol{\varepsilon}}=\varepsilon\overline{\mathbf{I}}_t+ig\mathbf{e}_z\times\overline{\mathbf{I}}_t+\varepsilon_z\mathbf{e}_z\mathbf{e}_z$ is the modified permittivity dyadic of a gyroelectric medium, taking into account the contributions due to the chirality. ξ_c and μ are the chiral parameter and permeability, respectively. Here, $\mathbf{I}_t=\mathbf{e}_x\mathbf{e}_x+\mathbf{e}_y\mathbf{e}_y$ stands for the transverse unit idem factor, and $\mathbf{e}_j$ denotes the unit vector in the j direction.

Substituting the constitutive relations (1a) and (1b) into the source-incorporated Maxwell's equations, the vector Helmholtz equation in the composite gyroelectric chiral medium is obtained [19,22]:

$$\nabla\times\nabla\times\mathbf{E}-2\omega\mu\xi_c\nabla\times\mathbf{E}-\omega^2\mu\overline{\boldsymbol{\varepsilon}}\cdot\mathbf{E}=i\omega\mu\mathbf{J}. \tag{2}$$

It should be noted that Eq. (2) satisfies the divergence equation $\nabla\cdot\overline{\boldsymbol{\varepsilon}}\cdot\mathbf{E}=\rho_e$, where ρ_e is the density of electric charge. If one takes the divergence of both sides of Eq. (2), we have $\nabla\cdot\overline{\boldsymbol{\varepsilon}}\cdot\mathbf{E}=-i\nabla\cdot\mathbf{J}/\omega$. Recalling the current continuity theorem $\nabla\cdot\mathbf{J}=i\omega\rho_e$, we see that Eq. (2) satisfies the divergence equation. Therefore, the divergence equation would not be taken into account in the following analysis in solving Eq. (2), since the solution of Eq. (2) would satisfy the divergence equation automatically. However, the continuity equation $\nabla\cdot\mathbf{J}=i\omega\rho_e$, which imposes a constraint condition on the source term of Eq. (2), must be taken into account when one tries to find a full solution to the source-incorporated Maxwell equations.

Noting the linearity of the vector identity (2), the electric field can be constructed from a three-dimensional transformation of current density $\mathbf{J}$ with respect to the dyadic Green's function $\overline{\boldsymbol{\Gamma}}(\mathbf{r},\mathbf{r}')$

$$\mathbf{E}=i\omega\mu\int_{V'}\mathbf{J}(\mathbf{r}')\cdot\overline{\boldsymbol{\Gamma}}(\mathbf{r},\mathbf{r}')dv', \tag{3}$$

where V' denotes the volume occupied by the exciting current source, and the current density $\mathbf{J}$ should satisfy the continuity equation $\nabla\cdot\mathbf{J}=i\omega\rho_e$ to ensure a reasonable solution to Eq. (2).

The differential equation, which the dyadic Green's function in the gyroelectric chiral medium $\overline{\boldsymbol{\Gamma}}(\mathbf{r},\mathbf{r}')$ must satisfy, can be obtained by substituting Eq. (3) into Eq. (2). Straightforward mathematical manipulation results in

$$\nabla\times\nabla\times\overline{\boldsymbol{\Gamma}}(\mathbf{r},\mathbf{r}')-2\omega\mu\xi_c\nabla\times\overline{\boldsymbol{\Gamma}}(\mathbf{r},\mathbf{r}')-\omega^2\mu\overline{\boldsymbol{\varepsilon}}\cdot\overline{\boldsymbol{\Gamma}}(\mathbf{r},\mathbf{r}')=\overline{\mathbf{I}}\delta(\mathbf{r}-\mathbf{r}'). \tag{4}$$

Here, $\overline{\mathbf{I}}$ and $\delta(\mathbf{r}-\mathbf{r}')$ denote unit dyadic and Dirac δ function, respectively. Since in the present investigation only Eq. (4) is concerned, the continuity equation $\nabla\cdot\mathbf{J}=i\omega\rho_e$, which imposes a constraint condition on the exciting source shall have nothing to do with the following formulations to derive the eigenfunction expansion of $\overline{\boldsymbol{\Gamma}}(\mathbf{r},\mathbf{r}')$.

In order to obtain the vector-wave-function-represented dyadic Green's function in an unbounded gyroelectric chiral medium that satisfies Eq. (4), we employ the well-known Ohm-Rayleigh method [3,4]. For this purpose, we first expand the source term of Eq. (4) in terms of the solenoidal and irrotational cylindrical vector wave functions [27] in a circular cylindrical coordinate system (ρ,ϕ,z):

$$\overline{\mathbf{I}}\delta(\mathbf{r}-\mathbf{r}')=\int_0^\infty dk_\rho\int_{-\infty}^\infty dk_z\sum_{n=-\infty}^{\infty}[\mathbf{A}_n(k_z,k_\rho)\mathbf{M}_n^{(1)}(k_z,k_\rho)+\mathbf{B}_n(k_z,k_\rho)\mathbf{N}_n^{(1)}(k_z,k_\rho)+\mathbf{C}_n(k_z,k_\rho)\mathbf{L}_n^{(1)}(k_z,k_\rho)], \tag{5}$$

where k_z,k_ρ are the spectral longitudinal and radial wave numbers, respectively. The vector expansion coefficients $\mathbf{A}_n(k_z,k_\rho)$, $\mathbf{B}_n(k_z,k_\rho)$, and $\mathbf{C}_n(k_z',k_\rho)$ are to be determined by employing the orthogonality relationships among the cylindrical vector wave functions. Here, the solenoidal and irrotational cylindrical vector wave functions are defined as [2,14]

$$\mathbf{M}_n^{(j)}(k_z,k_\rho)=\boldsymbol{\nabla}\times[\Psi_n^{(j)}(k_z,k_\rho)\mathbf{e}_z], \tag{6a}$$

$$\mathbf{N}_n^{(j)}(k_z,k_\rho)=\frac{1}{k_\lambda}\boldsymbol{\nabla}\times\mathbf{M}_n^{(j)}(k_z,k_\rho), \tag{6b}$$

$$\mathbf{L}_n^{(j)}(k_z,k_\rho)=\boldsymbol{\nabla}[\Psi_n^{(j)}(k_z,k_\rho)], \tag{6c}$$

where $k_\lambda=(k_\rho^2+k_z^2)^{1/2}$, and the generating function is [22]

$$\Psi_n^{(j)}(k_z,k_\rho)=Z_n^{(j)}(k_\rho\rho)e^{i(n+\phi+k_z z)}, \tag{6d}$$

with

$$Z_n^{(j)}(k_\rho\rho)=\begin{cases}J_n(k_\rho\rho), & j=1\\ Y_n(k_\rho\rho), & j=2\\ H_n^{(1)}(k_\rho\rho), & j=3\\ H_n^{(2)}(k_\rho\rho), & j=4,\end{cases} \tag{6e}$$

For the sake of self-consistency, the orthogonality relationships [27,28] of these vector wave functions can be summarized:

$$\int_{-\infty}^\infty\int_{-\infty}^\infty\int_{-\infty}^\infty\mathbf{M}_n^{(1)}(k_z,k_\rho)\cdot\mathbf{M}_{n'}^{(1)}(-k_z',k_\rho')dV=\int_{-\infty}^\infty\int_{-\infty}^\infty\int_{-\infty}^\infty\mathbf{N}_n^{(1)}(k_z,k_\rho)\cdot\mathbf{N}_{n'}^{(1)}(-k_z',k_\rho')dV=4\pi^2k_\rho\delta(k_\rho-k_\rho')\,\delta(k_z-k_z')\,\delta_{n(-n')}, \tag{7a}$$

$$\int_{-\infty}^\infty\int_{-\infty}^\infty\int_{-\infty}^\infty\mathbf{L}_n^{(1)}(k_z,k_\rho)\cdot\mathbf{L}_{n'}^{(1)}(-k_z',k_\rho')dV=\frac{4\pi^2(k_\rho^2+k_z^2)}{k_\rho}\,\delta(k_\rho-k_\rho')\,\delta(k_z-k_z')\,\delta_{n(-n')}, \tag{7b}$$

and

$$\int_{-\infty}^\infty\int_{-\infty}^\infty\int_{-\infty}^\infty\mathbf{M}_n^{(1)}(k_z,k_\rho)\cdot\mathbf{N}_{n'}^{(1)}(-k_z',k_\rho')dV=\int_{-\infty}^\infty\int_{-\infty}^\infty\int_{-\infty}^\infty\mathbf{N}_n^{(1)}(k_z,k_\rho)\cdot\mathbf{L}_{n'}^{(1)}(-k_z',k_\rho')dV=\int_{-\infty}^\infty\int_{-\infty}^\infty\int_{-\infty}^\infty\mathbf{L}_n^{(1)}(k_z,k_\rho)\cdot\mathbf{M}_{n'}^{(1)}(-k_z',k_\rho')dV=0, \tag{7c}$$

where $\delta_{n(-n')}$ is the Kronecker δ function (i.e., it is 1 for $n=-n'$ and 0 for $n\neq -n'$). With proper mathematical manipulation, we obtain the vector expansion coefficients of (5):

$$\mathbf{A}_n(k_z,k_\rho)=\frac{1}{4\pi^2k_\rho}\mathbf{M}_{-n}^{(1)\prime}(-k_z,k_\rho), \tag{8a}$$

$$\mathbf{B}_n(k_z,k_\rho)=\frac{1}{4\pi^2k_\rho}\mathbf{N}_{-n}^{(1)\prime}(-k_z,k_\rho), \tag{8b}$$

$$\mathbf{C}_n(k_z,k_\rho)=\frac{k_\rho}{4\pi^2(k_\rho^2+k_z^2)}\mathbf{L}_{-n}^{(1)\prime}(-k_z,k_\rho), \tag{8c}$$

where the primes over the cylindrical vector wave functions denote that they are evaluated at the source point $\mathbf{r}'$.

Then, let the dyadic Green's function in an unbounded gyroelectric chiral medium have the following full eigenfunction expansion in terms of the above defined cylindrical vector wave functions (the completeness property of the vector wave functions [28] guarantees such a representation)

$$\overline{\boldsymbol{\Gamma}}(\mathbf{r},\mathbf{r}')=\int_{-\infty}^\infty dk_z\int_0^\infty dk_\rho\sum_{n=-\infty}^{\infty}[\mathbf{a}_n(k_z,k_\rho)\mathbf{M}_n^{(1)}(k_z,k_\rho)+\mathbf{b}_n(k_z,k_\rho)\mathbf{N}_n^{(1)}(k_z,k_\rho)+\mathbf{c}_n(k_z,k_\rho)\mathbf{L}_n^{(1)}(k_z,k_\rho)], \tag{9}$$

where the vector expansion coefficients $\mathbf{a}_n(k_z,k_\rho)$, $\mathbf{b}_n(k_z,k_\rho)$, and $\mathbf{c}_n(k_z,k_\rho)$ can be obtained from Eqs. (4)–(8). Substituting Eqs. (9) and (5) into Eq. (4), which the dyadic Green's function must satisfy, and noting the instinct properties of the vector wave functions [2,14,28]

$$\mathbf{M}_n^{(1)}(k_z,k_\rho)=\frac{1}{k_\lambda}\boldsymbol{\nabla}\times\mathbf{N}_n^{(1)}(k_z,k_\rho), \tag{10a}$$

$$\mathbf{N}_n^{(1)}(k_z,k_\rho)=\frac{1}{k_\lambda}\boldsymbol{\nabla}\times\mathbf{M}_n^{(1)}(k_z,k_\rho), \tag{10b}$$

$$\boldsymbol{\nabla}\times\mathbf{L}_n^{(1)}(k_z,k_\rho)=0, \tag{10c}$$

we end up with

$$\left.\begin{matrix}\mathbf{a}_n(k_z,k_\rho)\\ \mathbf{b}_n(k_z,k_\rho)\end{matrix}\right\}=\tfrac{1}{2}[(k_\lambda^2-2\omega\mu\xi_c k_\lambda)\overline{\mathbf{I}}-\omega^2\mu\overline{\boldsymbol{\varepsilon}}]^{-1}\cdot[\mathbf{A}_n(k_z,k_\rho)+\mathbf{B}_n(k_z,k_\rho)]\pm\tfrac{1}{2}[(k_\lambda^2+2\omega\mu\xi_c k_\lambda)\overline{\mathbf{I}}-\omega^2\mu\overline{\boldsymbol{\varepsilon}}]^{-1}\cdot[\mathbf{A}_n(k_z,k_\rho)-\mathbf{B}_n(k_z,k_\rho)], \tag{11a}$$

$$\mathbf{c}_n(k_z,k_\rho) = -\frac{1}{\omega^2\mu}\,\overline{\boldsymbol{\epsilon}}^{-1}\cdot\mathbf{C}_n(k_z,k_\rho). \tag{11b}$$

It should be noted that the calculation that substituting Eqs. (9) and (5) into Eq. (4) gives Eq. (11) is based upon the condition that one can interchange the summation on n and the integrals on k_z, k_ρ. This condition can be justified if one notes that the terms in the square brackets of Eqs. (5) and (9) are continuous with respect to k_z and k_ρ, simultaneously.

To simplify the following analysis, we introduce a pair of linear combinations of the above defined solenoidal vector wave functions:

$$\left.\begin{matrix}\mathbf{V}_n^{(j)}(k_z,k_\rho)\\ \mathbf{W}_n^{(j)}(k_z,k_\rho)\end{matrix}\right\} = \frac{\mathbf{M}_n^{(j)}(k_z,k_\rho)\pm\mathbf{N}_n^{(j)}(k_z,k_\rho)}{\sqrt{2}} = \begin{cases}\mathbf{T}_v(k_\rho,k_z)[\Psi_n^{(j)}(k_z,k_\rho)]\\ \mathbf{T}_W(k_\rho,k_z)[\Psi_n^{(j)}(k_z,k_\rho)]\end{cases} \tag{12}$$

for $j=1,2,3,4$. Here, $\mathbf{T}_v(k_\rho,k_z)$ and $\mathbf{T}_w(k_\rho,k_z)$ denote the vector differential operators, whose explicit expressions can be straightforwardly obtained and will be omitted here. Taking physical insight into the introduced vector wave functions $\mathbf{V}_n^{(j)}(k_z,k_\rho)$ and $\mathbf{W}_n^{(j)}(k_z,k_\rho)$, we find that they correspond to the left- and right-handed circularly polarized eigenwaves of wave number $(k_z^2+k_\rho^2)^{1/2}$, respectively. Based on these combined vector wave functions, we can rewrite Eq. (9) as

$$\begin{aligned}\overline{\boldsymbol{\Gamma}}(\mathbf{r},\mathbf{r}') = \int_{-\infty}^{\infty} dk_z \int_0^\infty dk_\rho \sum_{n=-\infty}^{\infty} [&\mathbf{p}_n(k_z,k_\rho)\mathbf{V}_n^{(1)}(k_z,k_\rho)\\ &+\mathbf{q}_n(k_z,k_\rho)\mathbf{W}_n^{(1)}(k_z,k_\rho)\\ &+\mathbf{c}_n(k_z,k_\rho)\mathbf{L}_n^{(1)}(k_z,k_\rho)],\end{aligned} \tag{13}$$

where

$$\begin{aligned}\mathbf{p}_n(k_z,k_\rho) = \frac{1}{4\pi^2 k_\rho}[&(k_\lambda^2 - 2\omega\mu\xi_c k_\lambda)\overline{\mathbf{I}}\\ &-\omega^2\mu\overline{\boldsymbol{\epsilon}}]^{-1}\cdot\mathbf{V}_{-n}^{(1)\prime}(-k_z,k_\rho),\end{aligned} \tag{14a}$$

$$\begin{aligned}\mathbf{q}_n(k_z,k_\rho) = \frac{1}{4\pi^2 k_\rho}[&(k_\lambda^2 + 2\omega\mu\xi_c k_\lambda)\overline{\mathbf{I}}\\ &-\omega^2\mu\overline{\boldsymbol{\epsilon}}]^{-1}\cdot\mathbf{W}_{-n}^{(1)\prime}(-k_z,k_\rho),\end{aligned} \tag{14b}$$

and $\mathbf{c}_n(k_z,k_\rho)$ is given as Eq. (11b).

In this way, the dyadic Green's function in an unbounded gyroelectric chiral medium is explicitly represented in the form of the eigenfunction expansion in terms of the cylindrical vector wave functions, as given in Eq. (13). However, for practical applications and interpretation to possible novel phenomena, mathematical simplification to Eq. (13) is necessary, which will be reported in detail in the following analysis.

III. CONTRIBUTIONS FROM THE IRROTATIONAL VECTOR WAVE FUNCTIONS

In this section, we will simplify and give physical insight into the contributions from the irrotational vector wave functions to the dyadic Green's function. For the sake of convenience, we rewrite the irrotational part of the dyadic Green's function as

$$\begin{aligned}\overline{\boldsymbol{\Gamma}}_{\text{irr}}(\mathbf{r},\mathbf{r}') &= \int_{-\infty}^{\infty} dk_z \int_0^\infty dk_\rho \sum_{n=-\infty}^{\infty}[\mathbf{c}_n(k_z,k_\rho)\mathbf{L}_n^{(1)}(k_z,k_\rho)]\\ &= -\frac{1}{4\pi^2\omega^2\mu}\,\overline{\boldsymbol{\epsilon}}^{-1}\cdot\int_{-\infty}^{\infty} dk_z\int_0^\infty dk_\rho\\ &\quad\times\sum_{n=-\infty}^{\infty}\frac{k_\rho}{k_\lambda^2}\,\mathbf{L}_{-n}^{(1)\prime}(-k_z,k_\rho)\mathbf{L}_n^{(1)}(k_z,k_\rho).\end{aligned} \tag{15}$$

In Eq. (15), the integrals with respect to the k_z and k_ρ variables can be analytically evaluated by using the method of residues, similar to the procedure as outlined in detail in [29]:

$$\overline{\boldsymbol{\Gamma}}_{\text{irr}}(\mathbf{r},\mathbf{r}') = \frac{1}{4\pi\omega^2\mu}\,\overline{\boldsymbol{\epsilon}}^{-1}\cdot\nabla\nabla\left(\frac{1}{|\mathbf{r}-\mathbf{r}'|}\right). \tag{16}$$

We can also verify that Eq. (16) is correct. If we take the divergence of both sides of Eq. (4), we have, $-\omega^2\mu\overline{\boldsymbol{\epsilon}}\cdot\nabla\cdot\overline{\boldsymbol{\Gamma}}_{\text{irr}} = \nabla\delta(\mathbf{r}-\mathbf{r}')$. However, since $\nabla^2(1/|\mathbf{r}-\mathbf{r}'|) = -4\pi\delta(\mathbf{r}-\mathbf{r}')$, we see that Eq. (16) is identically true. Recalling the identity [30]

$$\nabla\nabla\left(\frac{1}{|\mathbf{r}-\mathbf{r}'|}\right) = -\frac{4\pi}{3}\,\delta(\mathbf{r}-\mathbf{r}')\overline{\mathbf{I}} + \frac{1}{r^5}\,(3\mathbf{R}\mathbf{R}-r^2\overline{\mathbf{I}}) \tag{17}$$

with $\mathbf{R}=\mathbf{r}-\mathbf{r}'$ and $r=|\mathbf{r}-\mathbf{r}'|$, we can rewrite Eq. (16) as

$$\overline{\boldsymbol{\Gamma}}_{\text{irr}}(\mathbf{r},\mathbf{r}') = -\frac{1}{4\pi\omega^2\mu}\left\{\left[\frac{4\pi}{3}\,\delta(\mathbf{r}-\mathbf{r}')+\frac{1}{r^3}\right]\overline{\boldsymbol{\epsilon}}^{-1} - \frac{3}{r^5}\,\overline{\boldsymbol{\epsilon}}^{-1}\cdot\mathbf{R}\mathbf{R}\right\}. \tag{18}$$

From Eq. (18), it is found that the singularity of the dyadic Green's function in a gyroelectric chiral medium, owing to the contributions from the irrotational vector wave functions, is $-\overline{\boldsymbol{\epsilon}}^{-1}\delta(\mathbf{r}-\mathbf{r}')/(3\omega^2\mu)$, which generalizes the counterpart of isotropic media [27,31]. Equation (18) also indicates that the irrotational vector wave functions in the full eigenfunction expansion of the dyadic Green's function contribute not only at source points, but also away from the source, similar to the case of an isotropic medium [32]. Another important conclusion we could draw from Eq. (18) is that the contributions from the irrotational vector wave functions to the dyadic Green's function in a gyroelectric chiral medium are essentially static in character, and independent of the chiral parameter ξ_c, which indicates that the interplay between gyroelectric and chiral parameters does not take effect for the nonpropagating waves. The fact that the divergence of the dyadic Green's function in a gyroelectric chiral medium does not depend on the chiral parameter leads to the results that the contributions from the irrotational part to the dyadic Green's function are independent of the chiral parameter as well.

IV. CONTRIBUTIONS FROM THE SOLENOIDAL VECTOR WAVE FUNCTIONS

In this section, we will analytically evaluate the k_ρ and k_z integrals for the solenoidal parts of the dyadic Green's function arisen in Eq. (13), respectively. This effort is intended to make the results presented in Sec. II applicable in solving the source-incorporated boundary value problems of cylindrically and planarly multilayered structures consisting of gyroelectric chiral media.

A. Analytical evaluation of the k_ρ integral

For simplicity, we rewrite the first term of Eq. (13) as

$$\overline{\mathbf{P}}=\frac{1}{4\pi^2}\int_{-\infty}^{\infty}dk_z\int_0^{\infty}\frac{dk_\rho}{k_\rho}\times\sum_{n=-\infty}^{\infty}\overline{\mathbf{A}}^{-1}\cdot\mathbf{V}_{-n}^{(1)'}(-k_z,k_\rho)\mathbf{V}_n^{(1)}(k_z,k_\rho), \tag{19}$$

where the explicit expression of the dyadic $\overline{\mathbf{A}}^{-1}$ are given in the Appendix.

Applying the technique apparently attributable to Sommerfeld [33] and the residue calculus through a modified contour with respect to the k_ρ integral, we have

$$\int_0^{\infty}dk_\rho\,\frac{\mathbf{V}_{-n}^{(1)'}(-k_z,k_\rho)\mathbf{V}_n^{(1)}(k_z,k_\rho)}{k_\rho(k_\lambda-k_1)(k_\lambda-k_2)}=\frac{i\pi}{(k_1-k_2)}\sum_{j=1}^{2}\frac{(-1)^{j+1}}{k_{\lambda j}}\mathbf{V}_m^{(3)>}(k_z,k_{\lambda j})\times\mathbf{V}_{-n}^{(1)<}(-k_z,k_{\lambda j})-\frac{1}{|n|(k_z-k_1)(k_z-k_2)}\times\mathbf{T}_\nu\mathbf{T}_{\nu'}\left[\left(\frac{\rho_<}{\rho_>}\right)^{|n|}\right]e^{i[n(\phi^>-\phi^<)+k_z(z^>-z^<)]}. \tag{20}$$

Here, $\rho_>=\max(\rho,\rho')$, $\rho_<=\min(\rho,\rho')$, $k_{\lambda j}=(k_j^2-k_z^2)^{1/2}$, and the superscripts $>$ and $<$ denote that the vector wave functions and variables (z and ϕ) are evaluated at $\mathbf{r}$ or $\mathbf{r}'$ corresponding to $\rho_>$ or $\rho_<$, respectively. It should be pointed out that the vector operators $\mathbf{T}_v$, $\mathbf{T}_{v'}$, $\mathbf{T}_w$, and $\mathbf{T}_{w'}$ used in this subsection are the abbreviated notations of $\mathbf{T}_v(k_\rho=0,k_z)$, $\mathbf{T}_{v'}(k_\rho=0,k_z)$, $\mathbf{T}_w(k_\rho=0,k_z)$, and $\mathbf{T}_{w'}(k_\rho=0,k_z)$, respectively. $\mathbf{T}_{v'}(k_\rho,k_z)$ and $\mathbf{T}_{w'}(k_\rho,k_z)$ can be separately obtained from the vector operators $\mathbf{T}_v(k_\rho,k_z)$ and $\mathbf{T}_w(k_\rho,k_z)$, but evaluated at the source point $\mathbf{r}'$. In Eq. (20) and in the following formulations, only the terms corresponding to $k_j>0$ are adopted for the summation. Also, we should recognize that the evaluation of Eq. (20) excludes the contributions from the $n=0$ term because the logarithmic singularity of the zeroth-order Hankel function at $k_\rho=0$ does not give rise to any contributions, as first pointed in [27] for isotropic media.

Using the result of Eq. (20), the integral with respect to the k_ρ variable in Eq. (19) can be analytically evaluated, which results in

$$\overline{\mathbf{P}}=\frac{1}{4\pi^2}\int_{-\infty}^{\infty}dk_z\sum_{n=-\infty}^{\infty}\overline{\mathbf{P}}^{n\rho}=\frac{1}{4\pi^2}\int_{-\infty}^{\infty}dk_z\sum_{n=-\infty}^{\infty}[P_{\rho\rho}^{n\rho}\mathbf{e}_\rho\mathbf{e}_\rho+P_{\rho\phi}^{n\rho}\mathbf{e}_\rho\mathbf{e}_\phi+P_{\rho z}^{n\rho}\mathbf{e}_\rho\mathbf{e}_z+P_{\phi\rho}^{n\rho}\mathbf{e}_\phi\mathbf{e}_\rho+P_{\phi\phi}^{n\rho}\mathbf{e}_\phi\mathbf{e}_\phi+P_{\phi z}^{n\rho}\mathbf{e}_\phi\mathbf{e}_z+P_{z\rho}^{n\rho}\mathbf{e}_z\mathbf{e}_\rho+P_{z\phi}^{n\rho}\mathbf{e}_z\mathbf{e}_\phi+P_{zz}^{n\rho}\mathbf{e}_z\mathbf{e}_z], \tag{21}$$

with

$$\left.\begin{matrix}P_{\rho\rho}^{n\rho}\\P_{\phi\phi}^{n\rho}\end{matrix}\right\}=\left.\begin{matrix}\mathbf{e}_\rho\\\mathbf{e}_\phi\end{matrix}\right\}\cdot\left\{\frac{\pi i}{2}\left[\sum_{j=1}^{2}\frac{(-1)^{j+1}\mathbf{V}_n^{(3)>}(k_z,k_{\lambda j})\mathbf{V}_{-n}^{(1)<}(-k_z,k_{\lambda j})}{(k_1-k_2)k_{\lambda j}}+\sum_{j=3}^{4}\frac{(-1)^{j+1}\mathbf{V}_n^{3>}(k_z,k_{\lambda j})\mathbf{V}_{-n}^{(1)<}(-k_z,k_{\lambda j})}{(k_3-k_4)k_{\lambda j}}\right]-\frac{\mathbf{T}_\nu\mathbf{T}_{\nu'}[(\rho_</\rho_>)^{|n|}]}{2|n|}\left[\frac{1}{(k_z-k_1)(k_z-k_2)}+\frac{1}{(k_z-k_3)(k_z-k_4)}\right]e^{i[n(\phi^>-\phi^<)+k_z(z^>-z^<)]}\right\}\cdot\left\{\begin{matrix}\mathbf{e}_\rho,\\\mathbf{e}_\phi,\end{matrix}\right. \tag{22a}$$

$$\left.\begin{matrix}P_{\phi\rho}^{n\rho}\\-P_{\rho\phi}^{n\rho}\end{matrix}\right\}=-\left.\begin{matrix}\mathbf{e}_\phi\\\mathbf{e}_\rho\end{matrix}\right\}\cdot\left\{\frac{\pi}{2}\left[\sum_{j=1}^{2}\frac{(-1)^{j+1}\mathbf{V}_n^{(3)>}(k_z,k_{\lambda j})\mathbf{V}_{-n}^{(1)<}(-k_z,k_{\lambda j})}{(k_1-k_2)k_{\lambda j}}-\sum_{j=3}^{4}\frac{(-1)^{j+1}\mathbf{V}_n^{(3)>}(k_z,k_{\lambda j})\mathbf{V}_{-n}^{(1)<}(-k_z,k_{\lambda j})}{(k_3-k_4)k_{\lambda j}}\right]-\frac{i\mathbf{T}_\nu\mathbf{T}_{\nu'}[(\rho_</\rho_>)^{|n|}]}{2|n|}\left[\frac{1}{(k_z-k_1)(k_z-k_2)}-\frac{1}{(k_z-k_3)(k_z-k_4)}\right]e^{i[n(\phi^>-\phi^<)+k(z-z^<)]}\right\}\cdot\left\{\begin{matrix}\mathbf{e}_\rho\\\mathbf{e}_\phi\end{matrix}\right. \tag{22b}$$

$$P_{\phi z}^{n\rho}=P_{z\phi}^{n\rho}=P_{\rho z}^{n\rho}=P_{z\rho}^{n\rho}=0, \tag{22c}$$

and

$$P_{zz}^{n\rho}=\mathbf{e}_z\cdot\left\{\pi i\sum_{j=5}^{6}\frac{(-1)^{j+1}\mathbf{V}_n^{(3)>}(k_z,k_{\lambda j})\mathbf{V}_{-n}^{(1)<}(-k_z,k_{\lambda j})}{(k_5-k_6)k_{\lambda j}}-\frac{\mathbf{T}_v\mathbf{T}_{v'}[(\rho_</\rho_>)^{|n|}]}{|n|(k_z-k_5)(k_z-k_6)}e^{i[n(\phi^>-\phi^<)+k_z(z^>-z^<)]}\right\}\cdot\mathbf{e}_z. \tag{22d}$$

Applying the same procedure as outlined above, the integral with respect to the k_ρ variable in the second term of Eq. (13) can be analytically evaluated as

$$\overline{\mathbf{Q}}=\frac{1}{4\pi^2}\int_{-\infty}^{\infty}dk_z\int_0^{\infty}\frac{dk_\rho}{k_\rho}\sum_{n=-\infty}^{\infty}[(k_\lambda^2+2\omega\mu\xi_ck_\lambda)\overline{\mathbf{I}}-\omega^2\mu\overline{\boldsymbol{\epsilon}}]^{-1}\cdot\mathbf{W}_{-n}^{(1)\prime}(-k_z,k_\rho)\mathbf{W}_n^{(1)}(k_z,k_\rho)=\frac{1}{4\pi^2}\int_{-\infty}^{\infty}dk_z\sum_{n=-\infty}^{\infty}\overline{\mathbf{Q}}^{n\rho}$$

$$=\frac{1}{4\pi^2}\int_{-\infty}^{\infty}dk_z\sum_{n=-\infty}^{\infty}[Q_{\rho\rho}^{n\rho}\mathbf{e}_\rho\mathbf{e}_\rho+Q_{\rho\phi}^{n\rho}\mathbf{e}_\rho\mathbf{e}_\phi+Q_{\rho z}^{n\rho}\mathbf{e}_\rho\mathbf{e}_z+Q_{\phi\rho}^{n\rho}\mathbf{e}_\phi\mathbf{e}_\rho+Q_{\phi\phi}^{n\rho}\mathbf{e}_\phi\mathbf{e}_\phi+Q_{\phi z}^{n\rho}\mathbf{e}_\phi\mathbf{e}_z+Q_{z\rho}^{n\rho}\mathbf{e}_z\mathbf{e}_\rho+Q_{z\phi}^{n\rho}\mathbf{e}_z\mathbf{e}_\phi+Q_{zz}^{n\rho}\mathbf{e}_z\mathbf{e}_z], \tag{23}$$

where the explicit expression of dyadic $\overline{\mathbf{Q}}^{n\rho}$ can be obtained from $\overline{\mathbf{P}}^{n\rho}$, with the substitution of $\mathbf{V}$ by $\mathbf{W}$, $\mathbf{T}_V$ by $\mathbf{T}_W$, $\mathbf{T}_{V'}$ by $\mathbf{T}_{W'}$, and ξ_c by $-\xi_c$, respectively.

The full eigenfunction expansion of the dyadic Green's function in an unbounded gyroelectric chiral medium is the summation of $\overline{\mathbf{\Gamma}}_{\rm irr}(\mathbf{r},\mathbf{r}')$, $\mathbf{P}$, and $\mathbf{Q}$. It can be easily examined that the expressions of Eqs. (21) and (23) unify those of chiral media [34], where $k_1=k_3=k_5$ and $k_2=k_4=k_6$. The results presented in this subsection include the contributions owing to the singularity at the origin of k_ρ plane, which have been overlooked in the past work for isotropic media, as first pointed out in [27]. It should also be pointed out that the eigenfunction expansion of the dyadic Green's function, as given in the present form, is useful to analyze the source-incorporated electromagnetic boundary value phenomena of cylindrically multilayered structures consisting of gyroelectric chiral media.

B. Analytical evaluation of the k_z integral

Analytically evaluating the k_z integrals arisen in Eqs. (19) and (23), we obtain

$$\overline{\mathbf{P}}=\frac{1}{4\pi^2}\int_0^{\infty}dk_\rho\sum_{n=-\infty}^{\infty}\overline{\mathbf{P}}^{nz}=\frac{1}{4\pi^2}\int_0^{\infty}dk_\rho\sum_{n=-\infty}^{\infty}[P_{\rho\rho}^{nz}\mathbf{e}_\rho\mathbf{e}_\rho+P_{\rho\phi}^{nz}\mathbf{e}_\rho\mathbf{e}_\phi+P_{\rho z}^{nz}\mathbf{e}_\rho\mathbf{e}_z+P_{\phi\rho}^{nz}\mathbf{e}_\phi\mathbf{e}_\rho+P_{\phi\phi}^{nz}\mathbf{e}_\phi\mathbf{e}_\phi$$
$$+P_{\phi z}^{nz}\mathbf{e}_\phi\mathbf{e}_z+P_{z\rho}^{nz}\mathbf{e}_z\mathbf{e}_\rho+P_{z\phi}^{nz}\mathbf{e}_z\mathbf{e}_\phi+P_{zz}^{nz}\mathbf{e}_z\mathbf{e}_z], \tag{24a}$$

$$\overline{\mathbf{Q}}=\frac{1}{4\pi^2}\int_0^{\infty}dk_\rho\sum_{n=-\infty}^{\infty}\overline{\mathbf{Q}}^{nz}=\frac{1}{4\pi^2}\int_0^{\infty}dk_\rho\sum_{n=-\infty}^{\infty}[Q_{\rho\rho}^{nz}\mathbf{e}_\rho\mathbf{e}_\rho+Q_{\rho\phi}^{nz}\mathbf{e}_\rho\mathbf{e}_\phi+Q_{\rho z}^{nz}\mathbf{e}_\rho\mathbf{e}_z+Q_{\phi\rho}^{nz}\mathbf{e}_\phi\mathbf{e}_\rho+Q_{\phi\phi}^{nz}\mathbf{e}_\phi\mathbf{e}_\phi+Q_{\phi z}^{nz}\mathbf{e}_\phi\mathbf{e}_z$$
$$+Q_{z\rho}^{nz}\mathbf{e}_z\mathbf{e}_\rho+Q_{z\phi}^{nz}\mathbf{e}_z\mathbf{e}_\phi+Q_{zz}^{nz}\mathbf{e}_z\mathbf{e}_z], \tag{24b}$$

where

$$P_{\rho\rho}^{nz}=\frac{\pi i}{2}\,\mathbf{e}_\rho\cdot\left[\frac{1}{k_\rho(k_1-k_2)}\sum_{j=1}^{3}(-1)^{j+1}\begin{Bmatrix}\mathbf{V}_{-n}^{(1)\prime}(-\sqrt{k_j^2-k_\rho^2},k_\rho)\mathbf{V}_n^{(1)}(\sqrt{k_j^2-k_\rho^2},k_\rho)\\ \mathbf{V}_{-n}^{(1)\prime}(\sqrt{k_j^2-k_\rho^2},k_\rho)\mathbf{V}_n^{(1)}(-\sqrt{k_j^2-k_\rho^2},k_\rho)\end{Bmatrix}+\frac{1}{k_\rho(k_3-k_4)}\right.$$
$$\left.\times\sum_{j=3}^{4}(-1)^{j+1}\begin{Bmatrix}\mathbf{V}_{-n}^{(1)\prime}(-\sqrt{k_j^2-k_\rho^2},k_\rho)\mathbf{V}_n^{(1)}(\sqrt{k_j^2-k_\rho^2},k_\rho)\\ \mathbf{V}_{-n}^{(1)\prime}(\sqrt{k_j^2-k_\rho^2},k_\rho)\mathbf{V}_n^{(1)}(-\sqrt{k_j^2-k_\rho^2},k_\rho)\end{Bmatrix}\right]\cdot\mathbf{e}_\rho\quad\begin{cases}\text{for } z\geqslant z'\\ \text{for } z\leqslant z'\end{cases}, \tag{25a}$$

$$P_{\phi\rho}^{nz}=-\frac{\pi}{2}\,\mathbf{e}_\phi\cdot\left[\frac{1}{k_\rho(k_1-k_2)}\sum_{j=1}^{2}(-1)^{j+1}\times\begin{Bmatrix}\mathbf{V}_{-n}^{(1)\prime}(-\sqrt{k_j^2-k_\rho^2},k_\rho)\mathbf{V}_n^{(1)}(\sqrt{k_j^2-k_\rho^2},k_\rho)\\ \mathbf{V}_{-n}^{(1)\prime}(\sqrt{k_j^2-k_\rho^2},k_\rho)\mathbf{V}_n^{(1)}(-\sqrt{k_j^2-k_\rho^2},k_\rho)\end{Bmatrix}-\frac{1}{k_\rho(k_3-k_4)}\right.$$
$$\left.\times\sum_{j=3}^{4}(-1)^{j+1}\begin{Bmatrix}\mathbf{V}_{-n}^{(1)\prime}(-\sqrt{k_j^2-k_\rho^2},k_\rho)\mathbf{V}_n^{(1)}(\sqrt{k_j^2-k_\rho^2},k_\rho)\\ \mathbf{V}_{-n}^{(1)\prime}(\sqrt{k_j^2-k_\rho^2},k_\rho)\mathbf{V}_n^{(1)}(-\sqrt{k_j^2-k_\rho^2},k_\rho)\end{Bmatrix}\right]\cdot\mathbf{e}_\rho\quad\begin{cases}\text{for } z\geqslant z'\\ \text{for } z\leqslant z'\end{cases}, \tag{25b}$$

$$P_{\phi z}^{nz}=P_{z\phi}^{nz}=P_{\rho z}^{nz}=P_{z\rho}^{nz}=0, \tag{25c}$$

$$P_{zz}^{nz}=\frac{\pi i}{k_\rho(k_5-k_6)}\,\mathbf{e}_z\cdot\sum_{j=5}^{6}(-1)^{j+1}\times\begin{Bmatrix}\mathbf{V}_{-n}^{(1)\prime}(-\sqrt{k_j^2-k_\rho^2},k_\rho)\mathbf{V}_n^{(1)}(\sqrt{k_j^2-k_\rho^2},k_\rho)\\ \mathbf{V}_{-n}^{(1)\prime}(\sqrt{k_j^2-k_\rho^2},k_\rho)\mathbf{V}_n^{(1)}(-\sqrt{k_j^2-k_\rho^2},k_\rho)\end{Bmatrix}\cdot\mathbf{e}_z\quad\begin{cases}\text{for } z\geqslant z'\\ \text{for } z\leqslant z'\end{cases}. \tag{25d}$$

$P^{nz}_{\phi\phi}$ and $P^{nz}_{\rho\phi}$ can be separately obtained from $P^{nz}_{\rho\rho}$ and $P^{nz}_{\phi\rho}$ with the replacement of $\mathbf{e}_\rho$ by $\mathbf{e}_\phi$, and $\mathbf{e}_\phi$ by $-\mathbf{e}_\rho$, respectively. The analytical expression of dyadic $\mathbf{Q}^{nz}$ can be obtained from $\mathbf{P}^{nz}$, with the substitution of $\mathbf{V}$ by $\mathbf{W}$, $\mathbf{T}_V$ by $\mathbf{T}_W$, $\mathbf{T}_{V'}$ by $\mathbf{T}_{W'}$, and ξ_c by $-\xi_c$, respectively.

Here, we have employed the identity

$$\int_{-\infty}^{\infty} dk_z \frac{\mathbf{V}^{(1)'}_{-n}(-k_z,k_\rho)\mathbf{V}^{(1)}_n(k_z,k_\rho)}{k_\rho(k_\lambda-k_1)(k_\lambda-k_2)} = \frac{\pi i}{k_\rho(k_1-k_2)} \sum_{j=1}^{2} (-1)^{j+1} \times \begin{Bmatrix} \mathbf{V}^{(1)'}_{-n}(-\sqrt{k_j^2-k_\rho^2},k_\rho)\mathbf{V}^{(1)}_n(\sqrt{k_j^2-k_\rho^2},k_\rho) & \text{for } z \geq z' \\ \mathbf{V}^{(1)'}_{-n}(\sqrt{k_j^2-k_\rho^2},k_\rho)\mathbf{V}^{(1)}_n(-\sqrt{k_j^2-k_\rho^2},k_\rho) & \text{for } z \leq z'. \end{Bmatrix} \tag{26}$$

In this case, the full eigenfunction expansion of the dyadic Green's function is the summation of $\mathbf{\Gamma}_{\mathrm{irr}}(\mathbf{r},\mathbf{r}')$, $\mathbf{P}$, and $\mathbf{Q}$. It should be mentioned that the components arising in Eqs. (24a) and (24b) can be reduced to the counterparts of chiral media [34], if we let $\varepsilon=\varepsilon_z$ and $g=0$ in the constitutive relations and noting $k_1=k_3=k_5$, and $k_2=k_4=k_6$. This type of eigenfunction representation of the dyadic Green's function can be used to analyze the source-incorporated electromagnetic boundary value problems of planarly multilayered structures consisting of gyroelectric chiral media.

Straightforward mathematical analysis reveals that for a dipole source parallel to the z axis, only $\mathbf{V}^{(1)'}_0$ and $\mathbf{W}^{(1)'}_0$ terms exist for the dyadic Green's function, while the dyadic Green's function of dipole sources perpendicular to the z axis contains only the $\mathbf{V}^{(1)'}_1$ and $\mathbf{W}^{(1)'}_1$ terms. Therefore, Sommerfeld integrals of dipole radiation in a gyroelectric chiral medium involve only those Sommerfeld integrals of dipole radiation in an isotropic medium [33,35]. So, various approximate, asymptotic, and numerical methods for Sommerfeld integrals [35] can be applied to study the electromagnetic resonance, radiation, propagation, and scattering phenomena of planarly multilayered gyroelectric chiral media.

C. Basic physical insight

From the explicit expressions of $\overline{\mathbf{P}}$ and $\overline{\mathbf{Q}}$, it is clear that the $\mathbf{e}_\rho\mathbf{e}_z$, $\mathbf{e}_z\mathbf{e}_\rho$, $\mathbf{e}_\phi\mathbf{e}_z$, and $\mathbf{e}_z\mathbf{e}_\phi$ components of the dyadic Green's function contributed from the solenoidal vector wave functions in a gyroelectric chiral medium automatically vanish, while the values of the $\mathbf{e}_\rho\mathbf{e}_\phi$ and $\mathbf{e}_\phi\mathbf{e}_\rho$ components are identical but with opposite signs. These mathematical properties of the dyadic Green's functions due to the contributions from the solenoidal parts indicate that the same ρ- and ϕ- directed electric currents in an unbounded gyroelectric chiral medium would generate the opposite values of the electric fields in the ϕ and ρ directions and do not excite electric field in the z direction while for the z-oriented electric current, only z-directed electric field can be excited. These physical properties of the dyadic Green's function result from the gyroelectric effects of the gyroelectric chiral medium. On the other hand, the eigenfunction expansion of the dyadic Green's function in terms of the introduced vector wave functions $\mathbf{V}^{(j)}_n(k_z,k_\rho)$ and $\mathbf{W}^{(j)}_n(k_z,k_\rho)$ indicates that due to the effects of optical activity [9,16–18,34] in gyroelectric chiral medium, two types of eigenwaves, i.e., the left- and right-handed circularly polarized waves traveling with different wave numbers, can be simultaneously excited. Similar to the case of electromagnetic waves propagating in a chiral medium [16–18], the same equation (2) holds valid for the electromagnetic waves of right- and left-handed circularly polarized types, traveling with different velocities. The fact that the wave numbers of these eigenwaves are jointly determined by all the constitutive parameters reveals the existence of the interplay between chiral and gyroelectric parameters, and makes it possible to manage the chiral effects with the introduction of a controllable gyroelectric parameter, in the sacrifice of introducing gyroelectric effects.

V. CONCLUSIONS

In the present contribution, the full eigenfunction expansion of the dyadic Green's function in an unbounded gyroelectric chiral medium is obtained in terms of the cylindrical vector wave functions, based on the Ohm-Rayleigh method. The present formulations, which are greatly facilitated by introducing a pair of linear combinations of the solenoidal cylindrical vector wave functions $\mathbf{V}^{(j)}_n(k_z,k_\rho)$ and $\mathbf{W}^{(j)}_n(k_z,k_\rho)$, generalize the canonical solutions of the dyadic Green's function for isotropic media [4,27,31,32], and recover the counterparts of gyroelectric and chiral media [34]. It is found that, due to the gyroelectric effects, the dyadic Green's function in a gyroelectric chiral medium has the nonreciprocal property (as described in detail in [36]), i.e., $\mathbf{\Gamma}(\mathbf{r},\mathbf{r}')\neq\mathbf{\Gamma}^{\mathrm{T}}(\mathbf{r}',\mathbf{r})$, with the superscript T denoting the transpose operator. The contributions from the irrotational vector wave functions to the dyadic Green's function, which are essentially static in character, are independent of the chiral parameter ξ_c and give rise to not only contributions at the source points, but away from the source as well. Due to the effects of optical activity [9,16–18,34], two types of eigenwaves, the left- and right-handed circularly polarized waves, can be simultaneously excited in the gyroelectric chiral medium, each traveling with a different wave number. The introduction of the off-diagonal constitutive parameter g not only leads to the nonreciprocal property of the composite gyroelectric chiral media, but also results in two novel characteristics of the dyadic Green's function contributed from the solenoidal vector wave functions: (1) the elimination of the $\mathbf{e}_\rho\mathbf{e}_z$, $\mathbf{e}_z\mathbf{e}_\rho$, $\mathbf{e}_\phi\mathbf{e}_z$, and $\mathbf{e}_z\mathbf{e}_\phi$ components, (2) the equality of the $\mathbf{e}_\rho\mathbf{e}_\phi$ and $\mathbf{e}_\phi\mathbf{e}_\rho$ components but with opposite signs. The fact that the wave numbers in the eigenfunction expansion of the dyadic Green's function are jointly determined by the gyroelectric and chiral parameters makes the chirality management possible by introducing of a controllable gyroelectric parameter to manage the wave numbers of the eigenwaves propagating in the gyroelectric chiral medium. Although the present formulations are somewhat cumbersome, which is inevitable due to the complexity of the material we have try to tackle, they can be verified by comparing its special forms with those of existing results (e.g., comparing the present singularity with that of isotropic media [27,31], and the contributions from the solenoidal vector wave functions with the counterparts of chiral media [34], good agreement can be obtained). It is believed that the present formulations would be useful both in simplifying the analysis of source-incorporated electromagnetic boundary value phenomena of cylindrically and planarly multilayered structures consisting of gyroelectric chiral media, and in understanding the physical process of the chirality management. Applications of the present formulations in studying the electromagnetic scattering, resonance, propagation, and radiation phenomena relevant to gyroelectric chiral media are under investigation, and will be reported in the near future.

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APPENDIX: EXPLICIT EXPRESSIONS OF $\overline{\mathbf{A}}^{-1}$

Comparing the first term of Eq. (13) with Eq. (19) and noting the explicit expression of $\mathbf{P}_n(k_z, k_\rho)$, we have

$$\begin{aligned}\overline{\mathbf{A}}^{-1}=&[(k_\lambda^2-2\omega\mu\xi_c k_\lambda)\overline{\mathbf{I}}-\omega^2\mu\overline{\boldsymbol{\varepsilon}}]^{-1}=A_{\rho\rho}\mathbf{e}_\rho\mathbf{e}_\rho+A_{\rho\phi}\mathbf{e}_\rho\mathbf{e}_\phi\\&+A_{\rho z}\mathbf{e}_\rho\mathbf{e}_z+A_{\phi\rho}\mathbf{e}_\phi\mathbf{e}_\rho+A_{\phi\phi}\mathbf{e}_\phi\mathbf{e}_\phi+A_{\phi z}\mathbf{e}_\phi\mathbf{e}_z+A_{z\rho}\mathbf{e}_z\mathbf{e}_\rho\\&+A_{z\phi}\mathbf{e}_z\mathbf{e}_\phi+A_{zz}\mathbf{e}_z\mathbf{e}_z.\end{aligned}\tag{A1}$$

Straightforward algebraic manipulation leads to

$$A_{\rho\rho}=A_{\phi\phi}=\frac{1}{2}\left[\frac{1}{(k_\lambda-k_1)(k_\lambda-k_2)}+\frac{1}{(k_\lambda-k_3)(k_\lambda-k_4)}\right],\tag{A2}$$

$$A_{\phi\rho}=-A_{\rho\phi}=\frac{i}{2}\left[\frac{1}{(k_\lambda-k_1)(k_\lambda-k_2)}-\frac{1}{(k_\lambda-k_3)(k_\lambda-k_4)}\right],\tag{A3}$$

$$A_{\rho z}=A_{z\rho}=A_{\phi z}=A_{z\phi}=0,\tag{A4}$$

and

$$A_{zz}=\frac{1}{(k_\lambda-k_5)(k_\lambda-k_6)},\tag{A5}$$

where $k_1, k_2, k_3, k_4, k_5, k_6$ are determined as

$$k_1=\omega[\mu\xi_c+\sqrt{\mu(\varepsilon-g)+\mu^2\xi_c^2}],\tag{A6}$$

$$k_2=\omega[\mu\xi_c-\sqrt{\mu(\varepsilon-g)+\mu^2\xi_c^2}],\tag{A7}$$

$$k_3=\omega[\mu\xi_c+\sqrt{\mu(\varepsilon+g)+\mu^2\xi_c^2}],\tag{A8}$$

$$k_4=\omega[\mu\xi_c-\sqrt{\mu(\varepsilon+g)+\mu^2\xi_c^2}],\tag{A9}$$

$$k_5=\omega(\mu\xi_c+\sqrt{\mu\varepsilon_z+\mu^2\xi_c^2}),\tag{A10}$$

$$k_6=\omega(\mu\xi_c-\sqrt{\mu\varepsilon_z+\mu^2\xi_c^2}).\tag{A11}$$

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